

Finite state transducers

Data Structures and Algorithms for Computational Linguistics III
(ISCL-BA-07)

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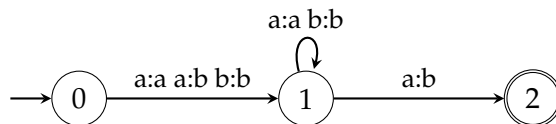
University of Tübingen
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Finite state transducers

A quick introduction

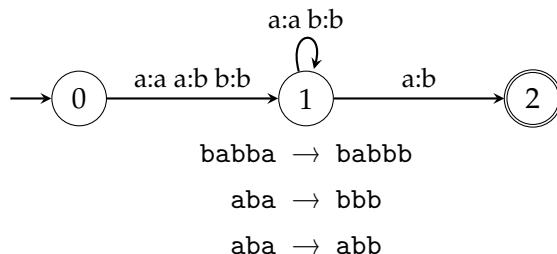
- A *finite state transducer* (FST) is a finite state machine where transitions are conditioned on pairs of symbols
- The machine moves between the states based on an *input* symbol, while it outputs the corresponding *output* symbol
- An FST encodes a *relation*, a mapping from a set to another
- The relation defined by an FST is called a *regular* (or *rational*) relation



Finite state transducers

A quick introduction

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Formal definition

A finite state transducer is a tuple $(\Sigma_i, \Sigma_o, Q, q_0, F, \Delta)$

Σ_i is the *input* alphabet

Σ_o is the *output* alphabet

Q a finite set of states

q_0 is the start state, $q_0 \in Q$

F is the set of accepting states, $F \subseteq Q$

Δ is a relation $(\Delta : Q \times \Sigma_i \rightarrow Q \times \Sigma_o)$

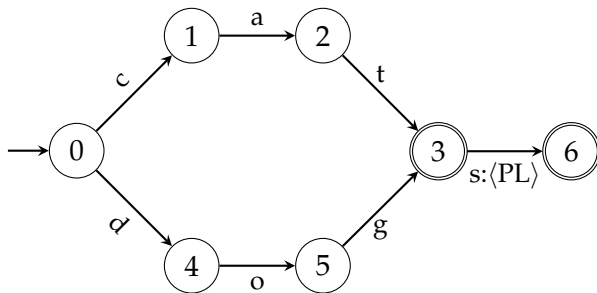
Where do we use FSTs?

Uses in NLP/CL

- Morphological analysis
- Spelling correction
- Transliteration
- Speech recognition
- Grapheme-to-phoneme mapping
- Normalization
- Tokenization
- POS tagging (not typical, but done)
- partial parsing / chunking
- ...

Where do we use FSTs?

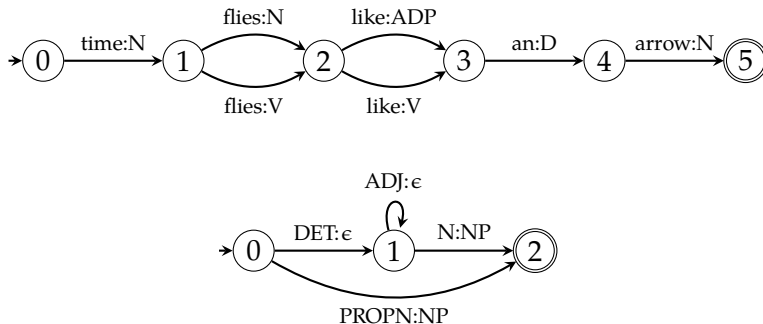
example 1: morphological analysis



In this lecture, we treat an FSA as a simple FST that outputs its input:
the edge label 'a' is a shorthand for 'a:a'.

Where do we use FSTs?

example 2: POS tagging / shallow parsing



Note: (1) It is important to express the ambiguity. (2) This gets interesting if we can 'compose' these automata.

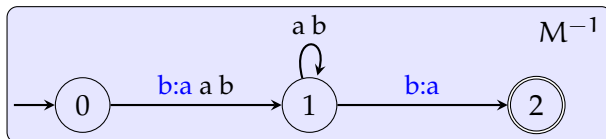
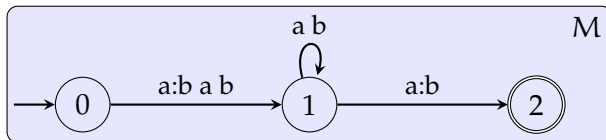
Closure properties of FSTs

Like FSA, FSTs are closed under some operations.

- Concatenation
- Kleene star
- Complement
- Reversal
- Union
- Intersection
- *Inversion*
- *Composition*

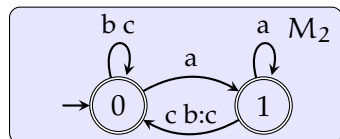
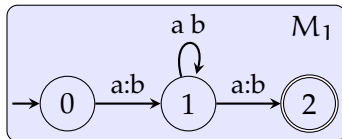
FST inversion

- Since an FST encodes a relation, it can be inverted
- Inverse of an FST swaps the input symbols with output symbols
- We indicate inverse of an FST M with M^{-1}



FST composition

sequential application



$aa \xrightarrow{M_1}$

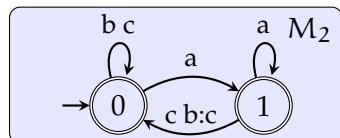
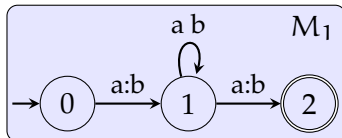
$bb \xrightarrow{M_1}$

$aaaa \xrightarrow{M_1}$

$abaa \xrightarrow{M_1}$

FST composition

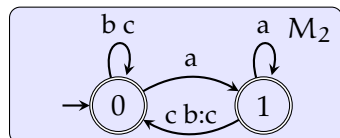
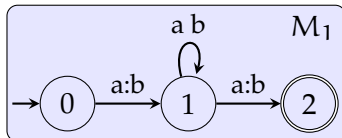
sequential application



aa	$\xrightarrow{M_1}$	bb	$\xrightarrow{M_2}$
bb	$\xrightarrow{M_1}$	\emptyset	$\xrightarrow{M_2}$
$aaaa$	$\xrightarrow{M_1}$	$baab$	$\xrightarrow{M_2}$
$abaa$	$\xrightarrow{M_1}$	$bbab$	$\xrightarrow{M_2}$

FST composition

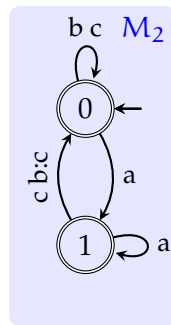
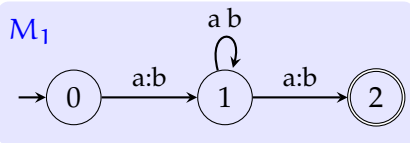
sequential application



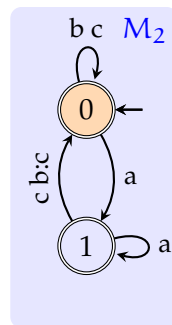
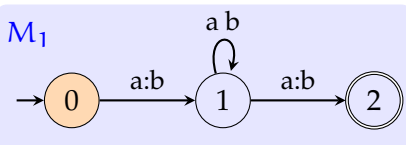
$M_2 \circ M_1$				
aa	$\xrightarrow{M_1}$	bb	$\xrightarrow{M_2}$	bb
bb	$\xrightarrow{M_1}$	\emptyset	$\xrightarrow{M_2}$	\emptyset
aaaa	$\xrightarrow{M_1}$	baab	$\xrightarrow{M_2}$	baac
abaa	$\xrightarrow{M_1}$	bbab	$\xrightarrow{M_2}$	bbac

- Can we compose two FSTs without running them sequentially?

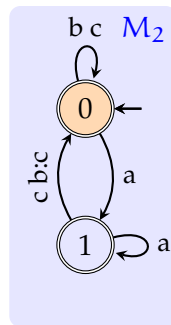
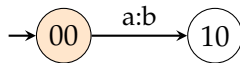
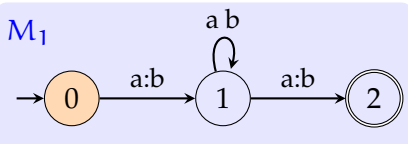
FST composition



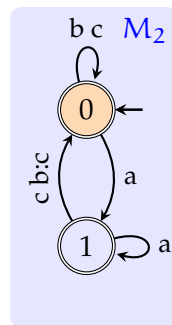
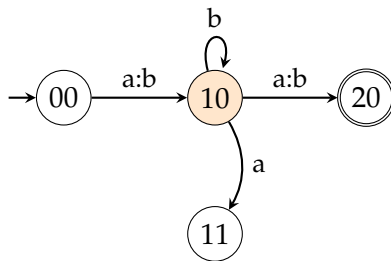
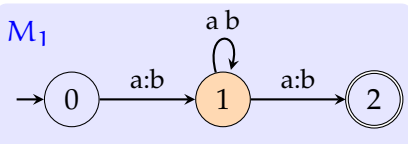
FST composition



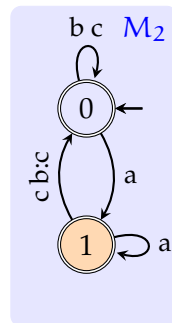
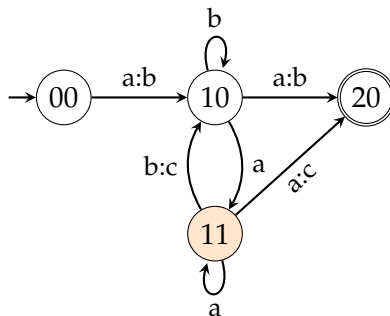
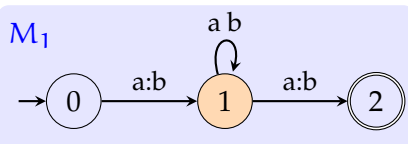
FST composition



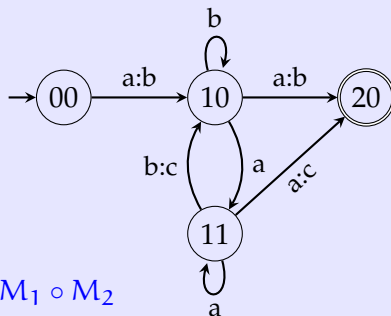
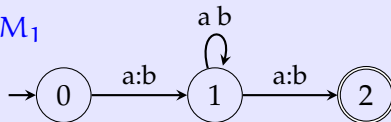
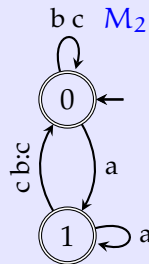
FST composition



FST composition

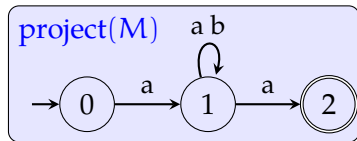
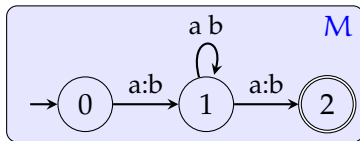


FST composition

 M_1

 $M_1 \circ M_2$

 M_2

Projection

- Projection* turns an FST into a FSA, accepting either the input language or the output language



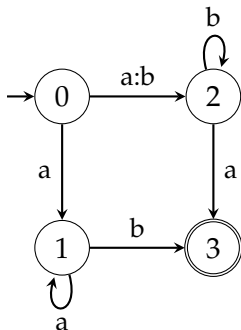
FST determinization

- A deterministic FST has unambiguous transitions from every state on any *input* symbol
- We can extend the subset construction to FSTs
- Determinization of FSTs means converting to a *subsequential* FST
- However, not all FSTs can be determinized

FST determinization

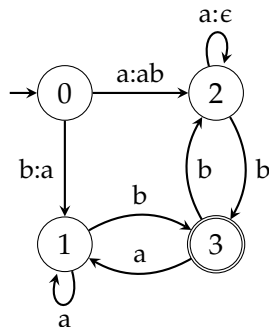
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Is this FST deterministic?



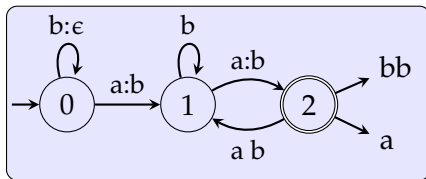
Sequential FSTs

- A sequential FST has a single transition from each state on every *input* symbol
- Output symbols can be strings, as well as ϵ
- The recognition is linear in the length of input
- However, sequential FSTs do not allow ambiguity



Subsequential FSTs

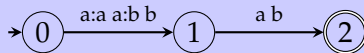
- A *k*-subsequential FST is a sequential FST which can output up to *k* strings at an accepting state
- Subsequential transducers allow limited ambiguity
- Recognition time is still linear



- The 2-subsequential FST above maps every string it accepts to two strings, e.g.,
 - $baa \rightarrow bba$
 - $baa \rightarrow bbbb$

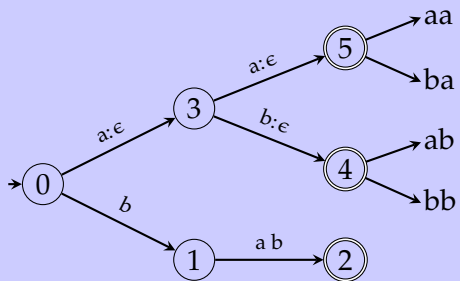
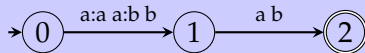
An exercise

Convert the following FST to a subsequential FST



An exercise

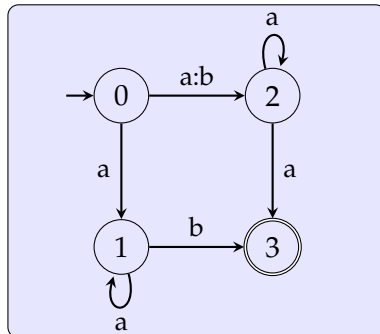
Convert the following FST to a subsequential FST



Determinizing FSTs

Another example

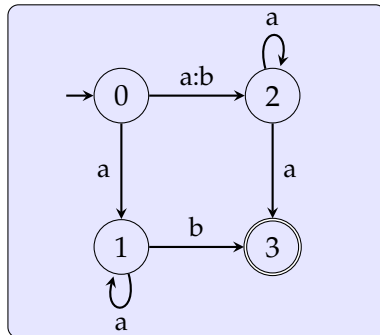
Can you convert the following FST to a subsequential FST?



Determinizing FSTs

Another example

Can you convert the following FST to a subsequential FST?



Note that we cannot ‘determine’ the output on first input until reaching the final input.

FSA vs FST

- FSA are *acceptors*, FSTs are *transducers*
- FSA accept or reject their input, FSTs produce output(s) for the inputs they accept
- FSA define sets, FSTs define relations between sets
- FSTs share many properties of FSAs. However,
 - FSTs are not closed under intersection and complement
 - We can compose (and invert) the FSTs
 - Determinizing FSTs is not always possible
- Both FSA and FSTs can be weighted (not covered in this course)





Next:

- FSA and regular languages
- Parsing

References / additional reading material

- Jurafsky and Martin (2009, Ch. 3)
- Additional references include:
 - Roche and Schabes (1996) and Roche and Schabes (1997): FSTs and their use in NLP
 - Mohri (2009): weighted FSTs

References / additional reading material (cont.)

-  Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*. second edition. Pearson Prentice Hall. ISBN: 978-0-13-504196-3.
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