NFA determinization

Data Structures and Algorithms for Computational Linguistics III (ISCL-BA-07)

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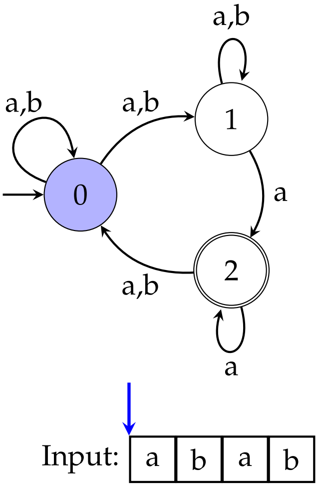
Recap

- Finite state automata come in two flavors
  - Deterministic (DFA): linear recognition time
  - Deterministic (NFA): sometimes more intuitive, easy to define, but exponential time (worst case) recognition
- The DFA and NFA are equivalent: for any language recognized by an NFA there is also a DFA recognizing the same language
- Then, the question is: how can we *determinize* an NFA to obtain an equivalent DFA

NFA recognition (again)



1. Start at q_0
2. Take the next input, mark all possible next states, repeat
3. If an accepting state is marked at the end of the input, accept

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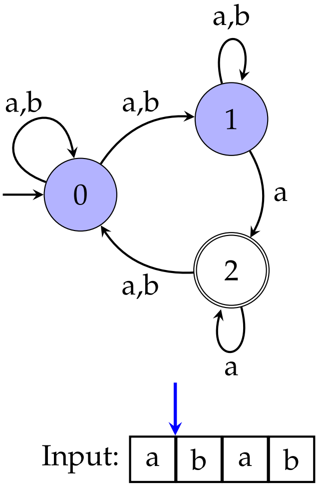

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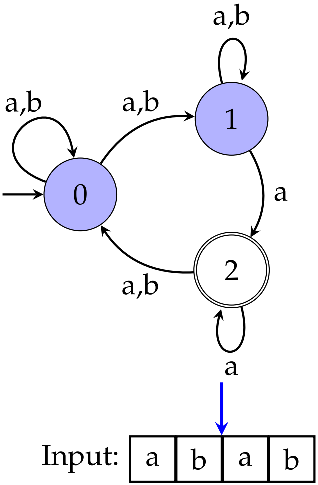

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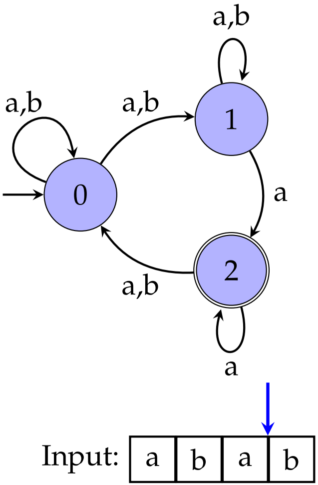

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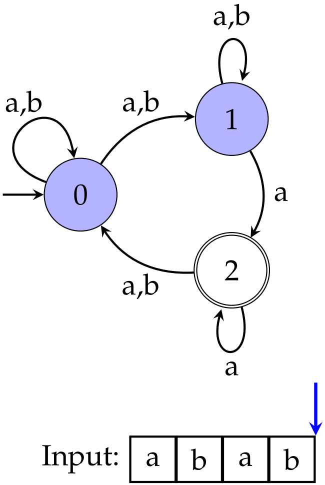

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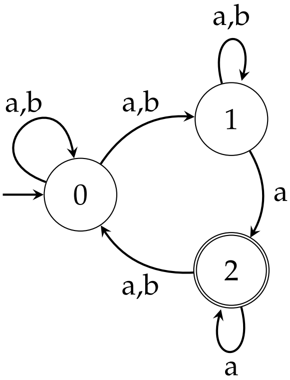


Input: | a | b | a | b |

1. Start at q_0
2. Take the next input, mark all possible next states, repeat
3. If an accepting state is marked at the end of the input, accept

The process is *deterministic*, and *finite-state*.

Determinization

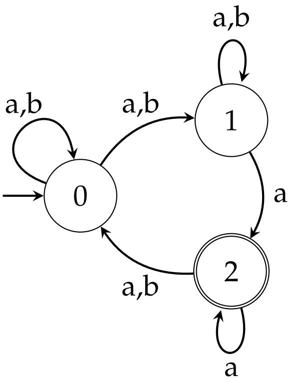
the subset construction

Intuition: remember the parallel NFA recognition. We can consider an NFA being a deterministic machine which is at a set of states at any given time.

- *Subset construction* (sometimes called power set construction) uses this intuition to convert an NFA to a DFA
- The algorithm can be modified to handle ϵ-transitions (or we can eliminate ϵ's as a preprocessing step)

The subset construction

by example



transition table with subsets

	symbol	
	a	**b**
$\emptyset$	$\emptyset$	$\emptyset$
$\rightarrow \{0\}$	$\{0, 1\}$	$\{0, 1\}$
$\{1\}$	$\{1, 2\}$	$\{1\}$
$* \{2\}$	$\{0, 2\}$	$\{0\}$
$\{0, 1\}$	$\{0, 1, 2\}$	$\{0, 1\}$
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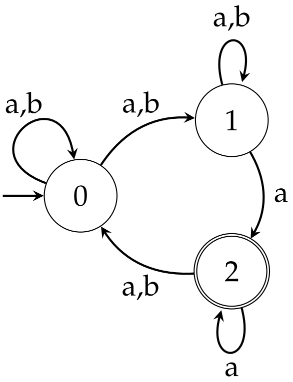

transition table with subsets

	symbol	
	a	**b**
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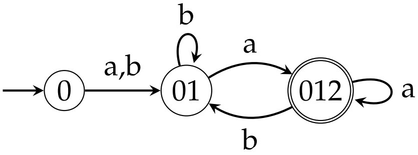
The subset construction

by example: the resulting DFA

transition table without useless/inaccessible states

	symbol a	b
$\rightarrow \{0\}$	$\{0, 1\}$	$\{0, 1\}$
$\{0, 1\}$	$\{0, 1, 2\}$	$\{0, 1\}$
$* \{0, 1, 2\}$	$\{0, 1, 2\}$	$\{0, 1\}$



The subset construction

by example: the resulting DFA

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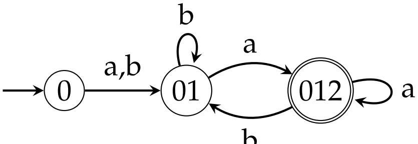
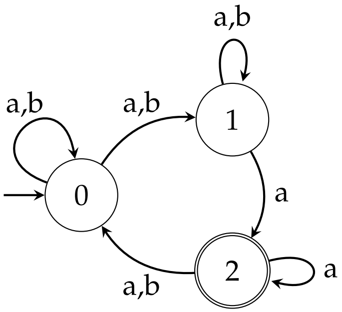


Do you remember the set of states marked during parallel NFA recognition?

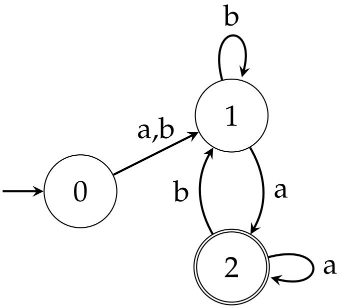
The subset construction

by example: side by side

NFA

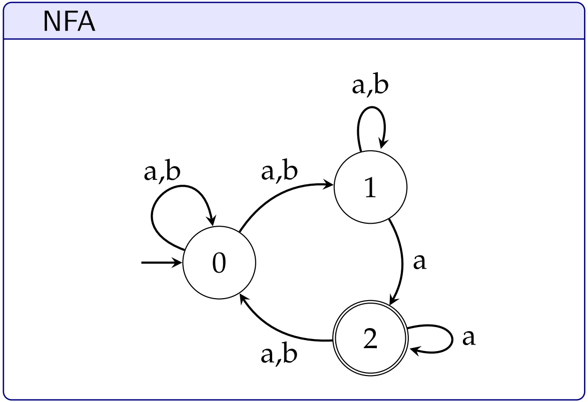


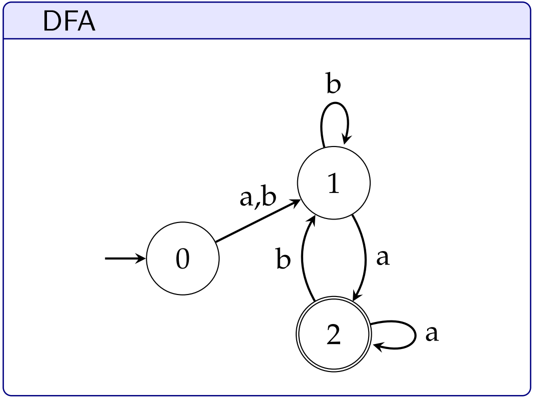
DFA



The subset construction

by example: side by side



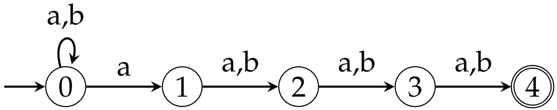


- What language do they recognize?

The subset construction

wrapping up

- In worst case, resulting DFA has 2^n nodes
- Worst case is rather rare, number of nodes in an NFA and the converted DFA are often similar
- In practice, we do not need to enumerate all 2^n subsets
- We've already seen a typical problematic case:

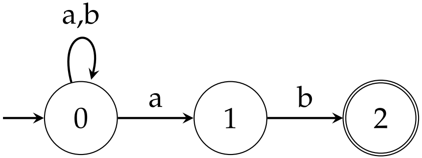


- We can also skip the unreachable states during subset construction

Yet another exercise

Determinize the following automaton

NFA:



Summary

- FSA are efficient tools with many applications
- FSA have two flavors: DFA, NFA (or maybe three: ε-NFA)
- DFA recognition is linear, recognition with NFA may require exponential time
- Reading suggestion: Hopcroft and Ullman (1979, Ch. 2&3), Jurafsky and Martin (2009, Ch. 2)

Next:

- Minimization

Acknowledgments, credits, references

- Hopcroft, John E. and Jeffrey D. Ullman (1979). *Introduction to Automata Theory, Languages, and Computation*. Addison-Wesley Series in Computer Science and Information Processing. Addison-Wesley. ISBN: 9780201029888.
- Jurafsky, Daniel and James H. Martin (2009). *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics, and Speech Recognition*. second edition. Pearson Prentice Hall. ISBN: 978-0-13-504196-3.

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